

# INTERMEDIATE MASS STARS: UPDATED MODELS

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## ABSTRACT

A new set of stellar models in the mass range 1.2 to 9  $M_{\odot}$  is presented. The adopted chemical compositions cover the typical galactic values, namely  $0.0001 \leq Z \leq 0.02$  and  $0.23 \leq Y \leq 0.28$ . A comparison among the most recent compilations of similar stellar models is also discussed. The main conclusion is that the differences among the various evolutionary results are still rather large. For example, we found that the H-burning evolutionary time may differ up to 20 %. An even larger disagreement is found for the He-burning phase (up to 40-50 %). Since the connection between the various input physics and the numerical algorithms could amplify or counterbalance the effect of a single ingredient on the resulting stellar model, the origin of this discrepancies is not evident. However most of these discrepancies, which are clearly found in the evolutionary tracks, are reduced on the isochrones. By means of our updated models we show that the ages inferred by the theory of stellar evolution is in excellent agreement with those obtained by using other independent methods applied to the nearby Open Clusters. Finally, the theoretical initial/final mass relation is revised.

*Subject headings:* stars: evolution – stars: intermediate mass – open clusters: general

## 1. INTRODUCTION

The comprehension of the evolutionary status of the various stellar systems, from the simplest stellar clusters up to the more complex galaxies, are mainly based on the comparison between theoretical stellar models and observational data. It follows that any improvement and/or assumptions in the basic input physics (equation of state, opacity, and the like) included in the computations of the stellar models directly influences the interpretation of the observed data. Moreover, since the population synthesis requires the availability of stellar models in a large range of masses and chemical compositions, and since such an homogeneous database is still missing, people involved in such kind of studies have been forced to collect models computed by different authors.

In this paper we present new evolutionary sequences for stellar masses ranging between 1.2 and 9  $M_{\odot}$ ; these models have been included in our database of stellar evolution which is available by anonymous ftp. This database also includes fully homogeneous models for low mass stars (Straniero, Chieffi & Limongi 1997) and those for massive stars (Chieffi, Limongi & Straniero, 1998, Limongi, Straniero & Chieffi, 1998). All these models have been obtained by means of the FRANEC, an acronym of Frascati Raphson Newton Evolutionary Code (Chieffi & Straniero, 1989, and Chieffi, Limongi & Straniero 1998). The input physics (equation of state, opacity, neutrino losses and the like) we are adopting at the moment are discussed in Straniero, Chieffi & Limongi (1997).

Several papers described sets of models of intermediate mass stars with various chemical compositions. However, owing to the continues improvement of the input physics, the (re)computation of the same stellar models becomes necessary. Major contributions are from: Kippenhahn, Thomas & Weigart 1965; Iben 1967a (and references therein); Paczynski 1970a, 1970b, 1971; Trimble, Paczynski & Zimmerman 1973; Alcock & Paczynski 1978; Becker & Iben 1979; Becker 1981; Vandenberg 1985; Maeder & Meynet, 1987, 1988, 1989,

1991; Castellani, Chieffi & Straniero 1990, 1992; Lattanzio, 1991; Stothers & Chin 1990, 1991a, 1991b, 1992; Vassiliadis & Wood 1992; Alongi et al. 1991, 1992; Bressan et al. 1993; Schaller et al. 1992. Thus, our new models will be compared with the most recent compilations of similar evolutionary sequences. Whenever possible, we analyze the origin of the resulting differences. The main goal of this study is to constraint the present level of uncertainty of the stellar evolution in the range of intermediate mass stars. In such a way, we provide a basic tool to check the reliability of our understanding of the galactic history as emerging from the study of population synthesis.

This is the plan of the paper: in the next section we revise the possible sources of uncertainties for H and He burning intermediate mass stellar models; in section 3 we describe our latest evolutionary computations for intermediate mass stars, from the ZAMS up to the AGB; in section 4 we discuss the comparisons among different evolutionary models; selected tests of the evolutionary sequences are presented in section 5. Final remarks follow.

## 2. INPUT PHYSICS AND CONVECTION

If the theoretical investigation of low mass stars appears well anchored to the result of helioseismology (see Straniero, Chieffi & Limongi, 1997) this is not the case for intermediate mass stars. It is commonly believed that the many uncertainties in the theory of turbulent convection still affects our understanding of the internal structure of these kind of stars. Owing to the lack of a conclusive test for the adequacy of the current theory of convection, the astrophysical literature presents a variety of different approaches to the computation of stellar models. It has been early recognized that the mixing of material in the core of a given star significantly alters its lifetime and in turn it could modify the age estimations of the various galactic components. The instability against turbulent convection is classically handled by means of a thermodynamical criterion (namely the Schwarzschild criterion for a

chemically homogeneous fluid). As it is well known, this criterion is based on the evaluation of the expected gradient of temperature produced by the radiative transport of energy: when the required gradient is too high, the radiative flux cannot account for the whole energy transport and hence convection is settled on.

First of all let us emphasize that the correct evaluation of the size of an unstable region is primarily dependent on the accuracy of the input physics. Any improvement of the stellar physics (eos, opacity, cross section and the like) could imply a variation of the estimated value of the temperature gradient and in turn it could modify the location of the borders of the convective regions, with sizable consequences on the computed stellar lifetime.

A second question concerns the possibility that the convective motion is not drastically inhibited in a stable region located just outside the convective core. In fact, although out of the Schwarzschild border a moving element of matter is subject to a strong deceleration, it might be possible that a non zero velocity is maintained for a certain path. In such a case this *mechanical overshoot* might induce a mixing of material in a radiatively stable region and might also contribute to the energy transport. A large number of papers have been devoted to the inclusion of such phenomenon in the computation of stellar models. Major contributions are from: Shaviv & Salpeter (1973); Maeder (1975); Cloutman & Whitaker (1980); Bressan et al (1981); Stothers & Chin (1981, 1990); Matraka, Wassermann & Weigert (1982); Xiong (1983, 1986); Doom (1982, 1985); Alongi et al. (1991); Maeder & Meynet (1987, 1988, 1989, 1991); Shaller et al. (1992).

Unfortunately the available convection theory is still inadequate for a reliable description of this phenomenon (see Renzini 1988 for a critical discussion on this argument), so that the evaluation of the degree of both matter and energy transport out of the unstable regions must be obtained by comparing the result of parameterized models with the measurements of some selected observable quantities (see e.g. Bressan et al. 1993). As for the *mixing length*

(Chieffi, Straniero & Salaris 1995), the calibration of the free parameters used to describe the mixing in the overshooting region, is model dependent. In fact, since the assumed input physics affects the size of the unstable regions, the calibration of the overshooting depends on these assumptions. For example, the larger is the opacity the larger is the convective region and in turn the lower is the required amount of overshooting. But the opacity coefficients are likely underestimated rather than overestimated. For this reason, if at the beginning of the eighties Becker & Methews (1983) claimed a relatively strong overshoot in order to reconcile the theory with the observed distribution of stars in the young Globular Clusters of the Magellanic Clouds, the latest attempts to derive the size of the convective core overshoot for H-burning stars indicate that, if it's present, it should be "mild" (see e.g. Sthothers & Chin 1992; Castellani, Chieffi & Straniero 1992; Schaller et al. 1992; Bressan et al. 1993; Demarque et al. 1994, Mermilliod et al. 1994; Schröder et al. 1997). In particular these studies generally found that the best reproduction of the various indicators of the convective core size is obtained with models including an overshoot roughly confined in between 0 and 0.3 (in units of pressure scale height and measured from above the stability border defined by means of the Schwarzschild criterion).

The situation is still more controversial for the central He-burning phase. In such a case, when He is converted into C within the core, the opacity increases and, in turn, the convective core size must increase (Schwarzschild 1970; Paczynski 1970b; Castellani et al. 1971a). As the He burning proceeds, a minimum in the radiative gradient settles on, so that mixing occurs in two separated regions: an internal one, which is fully convective, and an external one, in which the resulting mixture of C and He is just that needed to allow the convective neutrality (Castellani et al, 1971b; see also Iben 1986 and references therein). Such phenomenon was called *He burning semiconvection*. Close to the He exhaustion (namely when the central He becomes lower then approximately 0.1), some instabilities at the border of the convective core appears in stellar model computations (Castellani et al. 1985 a,b;

Iben 1986). These instabilities were called *breathing pulses* (BP). As a consequence of both semiconvection and BPs, a larger amount of fuel is available for the central He-burning. There is some debate concerning the actual occurrence of the BPs in real stars (Renzini & Fusi Pecci, 1988; Caputo et al., 1989). In any case, the inclusion of these phenomena might affect some important results of stellar evolution: the estimated central He-burning and AGB evolutionary times, the final amount of C and O in the core, the final WD mass. Note that the efficiency of both semiconvection and BPs also depend on the adopted input physics. For example, as firstly discussed by Iben (1972), the use of different prescriptions for the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  reaction rate alters the duration of the final part of the He-burning phase during which the BPs occur. Then, a larger value of this rate will enhance the effects of the BPs.

In summary, when comparing the evolutionary results obtained by different authors, the connection between the assumed mixing scheme and the adopted input physics must be taken into account. This will be done in section 4.

### 3. THE NEW MODELS

Models from 1.2 to 9  $M_{\odot}$  and metallicity ranging between  $10^{-4}$  and  $2 \times 10^{-2}$  have been computed from the ZAMS (Zero Age Main Sequence) up to the end of the E-AGB (Early Asymptotic Giant Branch) phase. The evolutionary tracks in the HR diagram are reported in figures 1, 2, 3 and 4. The runs of the central temperature versus the central density are shown in figures 5, 6, 7 and 8. Some examples of the evolution of the fully convective regions are illustrated in figures 9, 10, 11 and 12.

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In tables 1 to 4 we have reported the fundamental properties of the evolutionary sequences, namely from column 1 to 10: the total mass, the central H-burning lifetime (in Myr), the maximum size (in solar mass unit) of the convective core during central H-burning, the surface He mass fraction after the first dredge-up, the tip luminosity of the first RGB (red giant branch), the He core mass at the beginning of the He-burning, the central He-burning lifetime (in Myr), the He core mass (in solar unit) at the end of the He burning, the surface He mass fraction after the second dredge-up and the He core mass (in solar unit) at the beginning of the TP-AGB (thermally pulsing asymptotic giant branch) phase.



EDITOR: PLACE TABLE 1 HERE.

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In the following part of this section we briefly summarize the main features of the computed sequences of models, revising the dependence of the various evolutionary phases on the stellar mass and on the chemical composition. As already recalled in the introduction of this paper, the evolutionary history of an intermediate mass star, crossing the HR diagrams from the main sequence up to the AGB, is well known. For a more accurate description of the various evolutionary phases and an exhaustive list of references we remind the reader to the review paper by Iben (1991).

### **3.1. THE CENTRAL H-BURNING**

All the sequences of models having mass larger than  $1.2 M_{\odot}$  develop a convective core during the central H burning independently on the initial chemical composition. As a consequence, at variance with low mass stars for which the H burning occurs in a radiative environment, the evolutionary tracks evolve off the ZAMS towards lower temperature and larger luminosity. As the H is converted into He in the central region of the star, the opacity decreases and the convective core recedes (in mass). The convective instability in the core

is retained until the H mass fraction is reduced down to about 0.1. Then an overall contraction occurs and the star rapidly move towards the radiative main sequence. A maximum in luminosity is reached at the time of the H exhaustion.

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One interesting quantity characterizing the H-burning phase is the maximum extension of the convective core. As already recalled, this maximum is attained just after the ZAMS (see figures 9, 10, 11 and 12). In the last 30 years the computed values for this quantity have been systematically increased, mainly due to the increasing values of the adopted radiative opacity coefficients. Our present results are listed in column 3 of tables 1 to 4. Note that the size of the convective core decreases monotonically as the initial He increases while it initially increases as the metallicity decreases (up to  $Z=0.001$ ) and then it decreases at smaller metallicities. The corresponding H-burning evolutionary times are reported in column 2.

### **3.2. THE H-BURNING SHELL**

When the H-burning shell settles on, the convective envelope penetrates more than 80 % in mass of the star, bringing to the surface the products of the H burning. As firstly

pointed out by Iben (1964, 1967b), the main consequences of this first dredge-up are: the increase of the surface abundances of  $^4\text{He}$ ,  $^3\text{He}$  and  $^{14}\text{N}$  and a decrease of those of  $^{12}\text{C}$  and  $^{16}\text{O}$ . The modification of the surface composition is stronger in low mass stars due to the minor size of the envelope. The surface amount of He resulting after the first dredge-up in our models is listed in column 4 of tables 1 to 4. The subsequent evolution up to the onset of the He burning phase is mainly characterized by the equation of state governing the He core. In the strong degenerate regime a quite large He core mass is necessary to ignite He (namely about  $0.5 M_{\odot}$ , but it depends on the chemical composition). This is the case of a low mass star (i.e.  $M \leq 1.5 M_{\odot}$ ). For more massive stars the degree of degeneracy in the core is reduced and in turn the He ignition is more rapidly attained. In the asymptotic limit of non degenerate matter the minimum mass needed to ignite He is about  $0.35 M_{\odot}$ . For this reason the luminosity of the RGB tip and the He core mass attained at the He ignition (columns 5 and 6 of tables 1 to 4) decrease when the total mass increases from 1.5 up to  $2.5 M_{\odot}$ . When the stellar mass is larger than  $2.5\text{--}3 M_{\odot}$ , the off main sequence evolution is not further controlled by the growth of the He core. In such a case, owing to the internal mixing occurring during the main sequence, the H-burning shell at the beginning of the RGB settles well outside the minimum mass needed to ignite He. Hence, the RGB tip and the mass of the He core at the He ignition rise as the total mass increases.

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The minimum resulting by the combination of these two behaviors marks the so called RGB phase transition (Iben 1967c). Such an occurrence is illustrated in figure 13. According to the classical results, we found that the minimum core mass is attained for a total mass of  $2.3\text{--}2.5 M_{\odot}$ , value slightly increasing with the metallicity. Note that the almost constant minimum core mass at very different metallicities is the consequence of the opposite influence

of  $Z$  and  $Y$  on this quantity. In fact, as pointed out by Sweigart, Greggio & Renzini (1990), the transition mass increases as the metallicity increases and decreases as the He increases.

### 3.3. THE CENTRAL HE-BURNING

As it is well known, the larger the core mass at the He ignition the brighter is the star during the central Helium burning phase and the shorter is the central He burning lifetime. Hence a maximum in such a lifetime is expected, corresponding to the minimum He core mass occurring at the RGB phase transition. In figure 14 we have reported this lifetime for our models with  $Z=0.02$  and  $Y=0.28$ . They are listed in column 7 of tables 1 to 4 for the full set of models.

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During central He burning the evolutionary tracks move towards the blue part of the HR diagrams on a Kelvin-Helmoltz time scale and then moves back to the red giant branch when the central He vanishes. The extension of this loop depends on both the stellar mass and the chemical composition (see Alcock & Paczynski, 1978, and references therein). The larger the mass the hotter the left border of the loop, but an interesting exception is worth to be noted (Castellani, Chieffi & Straniero, 1990). At the lower metallicities ( $Z=0.0001$ ,  $0.001$  and  $0.006$ ), the more massive stars ignite He before they can reach the RGB, thus skipping the first dredge-up. It occurs for stellar masses  $M \geq 2.7M_{\odot}$ ,  $M \geq 4M_{\odot}$  and  $M \geq 5M_{\odot}$ , at  $Z=0.0001$ ,  $0.001$  and  $0.006$  respectively. Since the larger is the surface He amount the lower is the opacity, the fate of the first dredge-up affects the blue loop extension: those models in which the first dredge up do not occur have a narrower blue loop. Such an occurrence is clearly shown in figure 2, which reports the HR diagrams for  $Z=0.006$ : note that the He

burning evolutionary track of the  $4M_{\odot}$  sequence have a significantly larger blue loop than the  $5M_{\odot}$  one.

### 3.4. THE DOUBLE SHELL PHASE

During the whole central He burning phase the H-burning shell moves outward so that the longer is the central He burning lifetime the greater will be the increment of the He core mass (see column 8 in tables 1 to 4). For this reason, the minimum in the  $M_{He}$ -initial mass relation, which is evident at the beginning of the central He burning, is smoothed away at the end of the E-AGB phase (see figures 15). As a consequence all the stars with  $M \leq 3 M_{\odot}$  starting the thermally pulsing AGB phase have a rather similar He core mass, namely  $0.55 \pm 0.05 M_{\odot}$  depending on the metallicity (column 10 of tables 1 to 4). Such a value provides us a lower limit to the expected mass of a CO WD.

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For a sufficiently high initial stellar mass a second dredge-up occurs during the early AGB (Kippenhahn et al. 1965, Paczynski 1970, Becker & Iben 1979). In such a case, the convective envelope penetrates the H discontinuity located just below the H-burning shell so that the resulting He core mass is lowered with respect to the value attained at the end of the central He-burning (column 10 and 8 of tables 1 to 4, respectively). The second dredge-up takes place only if the H-burning shell extinguishes. This is not the case for a low mass stars, in which the expansion induced by the He-burning shell did not induce a sufficient cooling of the H-burning one. We found that the minimum mass for the occurrence of the second dredge-up is  $4 M_{\odot}$  for the three lower metallicities and a bit larger (about  $5 M_{\odot}$ ) in the case of  $Z = 0.02$ . The surface He mass fraction after the second dredge-up is listed in column 9 of tables 1 to 4.

### 3.5. THE FINAL MASSES AND $M_{UP}$

The white dwarf (WD) masses resulting from the evolution of low and intermediate mass stars are very important quantities for the purpose of the study of population synthesis, Planetary Nebula, Novae, Super novae, and the like (see e.g. Iben, 1991).

In figure 16 we compare our theoretical final masses with the relation reported by Weidemann (1987) and the updated one by Herwig (1995). Squares represent the He core masses at the end of the E-AGB, while arrows show the growth of the He core masses during the TP-AGB phase, as derived by using our thermally pulsing models (Straniero et al. 1996). The number reported at the top of each arrow is the number of thermal pulses computed up to the end of the AGB phase. It was determined according to the mass loss rate prescriptions of Groenewegen & de Jong (1994).

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Note that the Weidemann predictions (as well as the recent improvements incorporated by Herwig) are based on a semi empirical approach. They make use of various methods to evaluate the WD masses in nearby Open Clusters, whose initial mass is derived from the turnoff age. Our final masses are instead the result of a pure theoretical calculation and, hence, they are mainly dependent on the adopted input physics. Such a difference should be well kept in mind when comparing our results with those of Weidemann (or Herwig), as we do in figure 16. Despite the two different approaches, there is an acceptable agreement between our theoretical final masses and those of Weidemann (Herwig). In the next section we will discuss our final core masses in comparison with the ones obtained by other authors by means of alternative stellar evolutionary codes.

Stars with higher masses ignite the Carbon before the onset of the thermally pulsing phase (Paczynski 1971; Alcock & Paczynski 1978; Becker & Iben 1979,1980; Castellani,

Chieffi & Straniero 1991; Bressan et al. 1993; García-Berro, Ritossa, Iben 1997). In tables 1 to 4 we have distinguished among the models which experience an off center Carbon ignition and those with a central Carbon ignition. We found that  $M_{up}$ , i.e. the maximum mass for which the concurrent action of the pressure of a strong degenerate electron component and the neutrino energy loss in the core prevent the onset of the C-burning, ranges between 6.5 and 8  $M_{\odot}$ , the lower and the larger values being obtained for  $Z=0.0001$  and 0.02, respectively

#### 4. THE PRESENT LEVEL OF UNCERTAINTY

Owing to the large amount of numerical algorithms and physical ingredients commonly used in the computation of stellar models, the evaluation of their reliability is not trivial. A first idea of the possible sources of uncertainties can be obtained by comparing the evolutionary sequences obtained by different authors by means of different evolutionary codes and/or input physics. This might be also useful to evaluate the correctness of merging different sets of stellar models. As already recalled, there exist a rather large number of papers which present set of models for intermediate mass stars. In the following we will compare our results with the most recent and widely adopted collections of these stellar models.

##### 4.1. OLD AND NEW PHYSICS

Let us firstly compare the present computations to the ones of Castellani, Chieffi & Straniero (1990 and 1992), which were obtained by means of almost the same evolutionary code, but by adopting an "old" physics. Such a comparison will provide us with an evaluation of the importance of the most recent (last decade) improvements of the input physics. In figure 17 we have compared the evolutionary tracks for  $Z=0.02$ . The two sets of models appear rather similar except for some (important) details. Concerning the main sequence,

the new tracks are slightly brighter and the convective path (i.e. from the ZAMS up to the beginning of the overall contraction) is longer.

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The new H-burning lifetimes are generally lower ( $\sim 5\%$ ) than the old ones, but this is mainly due to the slightly lower amount of He used in our old computations (namely  $Y=0.27$ ). Concerning the He burning, the most striking difference is the extension of the blue loop in the more massive sequences, the new ones being significantly wider. The He burning lifetime is substantially unchanged.

#### 4.2. DIFFERENT EVOLUTIONARY CODES

The second step in the evaluation of the reliability of the evolutionary sequences will be the comparison with the most recent and widely adopted compilations of stellar models. Let us distinguish between models with and without mechanical convective core overshoot.

The most recent sets of intermediate mass stellar models without overshoot have been published by Lattanzio (1991, L91) and Vassiliadis and Wood (1993, VW93). Despite the differences in the chemical composition and in the input physics there is a good agreement between our H-burning models and the ones found in the two papers cited above. For example, by interpolating on the grid published by Lattanzio we derive for a  $2.5M_{\odot}$  ( $Z=0.02$  and  $Y=0.28$ ) an H-burning lifetime of 512 Myr to be compared with our result, namely 505 Myr. For the same stellar mass, but  $Y=0.25$ , Vassiliadis and Wood found 619 Myr. Since in this range of mass and metallicity we found that by increasing the original helium of  $\delta Y = 0.1$  the corresponding  $t_H$  must be reduced of about 25 Myr, the quoted value correspond to about 545 Myr at  $Y=0.28$ . Similar differences are found for other masses



and other chemical compositions. Concerning the He burning models the situation is more complicated. The He burning lifetime is strongly dependent on both the assumed scheme for convection and the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  reaction rate. As in VW93, we allow semiconvection and suppress breathing pulses, whereas they are both allowed in the Lattanzio’s computation. On the other hand, we use the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  reaction rate of Caughlan et al. (1985) which is about three time larger than the rates adopted by Lattanzio (1991) and Vassiliadis and Wood (1993). We recall that the larger this reaction rate the larger the He-burning lifetime. Bearing in mind these differences in the input physics, for the  $5M_{\odot}$  with  $Z=0.02$  (0.016 in VW93) one find 20.8, 23.5 and 30.6 in the present paper, VW93 and L91, respectively. For lower masses and/or metallicity the differences are similar. For example, for  $M = 1.5M_{\odot}$  and  $Z=0.001$  we found  $t_{He} = 97.4$  Myr to be compared to 122.2 Myr (VW93) and 118.3 (L91). In summary, our H-burning lifetimes appear in good agreement with those obtained in other studies, being the differences in the evolutionary time scales always lower than 10%. Note that similar differences were found with respect to our old computations. On the contrary, the present uncertainty in the theoretical evaluation of the He-burning lifetime is definitely larger. Differences up to 30% are found in  $t_{He}$ . In principle they should be primarily attributed to the uncertainties in the convective algorithm and/or in the major He burning reaction rates. In practice, due to the connection between input physics and mixing efficiency, it is rather complicated to disclose the origin of such differences.

Recent models including a moderate amount of convective core overshoot have been published in a series of papers by the Padua group (see Bressan et al., 1993, and references therein; in the following B93) and by the Geneve one (Schaller et al. 1992 and reference therein; in the following S92). We recall that the B93 models were obtained by extending, in practice, the mixed central region of an H-burning or He burning intermediate mass star by approximately  $0.25 H_p$  over the unstable zone, while Schaller et al. assume  $0.2 H_p$ .

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Concerning the H-burning, when the convective core overshoot is taken into account, a larger amount of fuel is available in the burning region of the star. However, since the larger is the mixed region the brighter is the star, this additional fuel is more rapidly burned, so that the effect of the core overshoot on the H- burning lifetime is partially counterbalanced. In figure 18 we compare our H-burning lifetimes to those resulting from the B93 and S92 models. As expected, overshoot models are generally older than the corresponding classical ones. Note, however, that despite the similar amount of overshoot assumed by Bressan et al. and Schaller et al., the differences between these two sets of models are comparable to the ones found with respect to our (no overshoot) models.

Another important consequence of the convective core overshoot during the central H-burning is the reduction of the mass at which the RGB transition occurs. Because the RGB evolution is faster if the star has a non degenerate He core, the number of stars lying on the RGB of galactic open clusters might be used, in principle, to derived the value of the transition mass and, in turn, to discriminate in between models with and without core overshoot (see e.g. Mermilliod et al., 1994 and references therein). The comparison between our models and those of Padua, shows that the difference is presently quite small. For example, at  $Z=0.02$  we found a *transition* mass of  $2.4 M_{\odot}$  while Bressan et al. (1993) found  $2.2 M_{\odot}$ . Thus, minor differences are aspected in the synthetic RGB populations. Note that a similar comparison cannot be made with the Schaller et al. models because their set is not spaced enough in mass.

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At variance with the H-burning phase, the inclusion of a moderate overshoot in computing He burning models do not significantly alter the resulting He-burning lifetime. In

fact, if a moderate overshoot is taken into account, the semiconvective layer is hidden by this extra mixing, but the total amount of fuel (He) available for the central nuclear burning should be practically the same than that found in models without core overshoot but including semiconvection. Our He-burning evolutionary times and those obtained by Bressan et al. and Schaller et al. are compared in figure 19. Note that B93 adopt the rather low Caughlan & Fowler (1988) rate for the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  reaction, whereas S92 use our preferred rate (i.e. Caughlan et al., 1985). The differences in the stellar lifetimes are larger than those found in the case of the H-burning phase (up to 60%). Also in this case, the origin of the disagreement is not easily recognized.

Let us conclude by noting that the typical differences which we found when comparing our models (no overshoot) with those by L91, VW93, B93 and S92 are of the same order of magnitude of those found in the comparisons between the two set of models with convective core overshoot. In other words, in the range of intermediate mass stellar models, the current uncertainty due to a possible not negligible occurrence of a convective core overshoot appears less severe, or of the same order of magnitude, than those induced by other input physics.

### 4.3. THE RELIABILITY OF THE THEORETICAL CORE MASSES

In the previous section we have compared the final masses obtained by evolving our models up to the end of the AGB to the semiempirical initial/final mass relation (Weidemann, 1987). These quantities depend on the core mass attained at the beginning of the thermally pulsing AGB phase and on the AGB mass loss rate (see e.g. Iben & Renzini 1983). For the more massive stars also the efficiency of the second dredge-up should be taken into account (Paczynski 1971, Becker & Iben 1979,1980) In the following we compare our evolutionary core masses at the first TP with the ones obtained by other authors. Let us recall that the larger the duration of the He burning phase the larger is the time available for the

shell H-burning to advance in mass. Hence, the large uncertainty on the current estimation of the stellar lifetime (as illustrated in section 4.2) might affects the theoretical previsions of the final masses. Concerning the convective algorithm, the lowest lifetime, and in turn the smallest He core mass, is obtained when semiconvection, BPs and overshooting are neglected, while an approximate doubling of the He-burning lifetime is found when, as in our models, only the semiconvection is taken into account. However by comparing the RGB, HB and AGB theoretical lifetime ratios to the observed stellar number ratios of well studied galactic Globular Clusters, it is possible to discriminate among the various mixing hypothesis (see e.g. Renzini & Fusi Pecci 1988). In such a way, there is a support to the classical semiconvection scheme (no BPs), but a moderate overshooting, which could mimic the effect of semiconvection, cannot be ruled out. The current uncertainty in the resonant contribution to the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  reaction rate might also change the estimated He burning lifetime (Iben 1967). In such a case, by varying the astrophysical factor in the range of value compatible with the available measurements for this reaction rate (see Buchmann, 1996 and 1997), we have obtained a variation of the He burning lifetime of about 5-10 %.

EDITOR: PLACE FIGURE 20 HERE.

In figure 20 we compare our core masses at the end of the E-AGB to those computed by Lattanzio (1991) and Bressan et al (1993). In spite of the rather large discrepancies found in the He-burning evolutionary time scales, there is a good agreement between the core masses obtained by the different authors. Only the core masses of the more massive models by Bressan et al. are rather larger than ours (i. e. for  $M \geq 5M_{\odot}$ ). From table 5 of B93 we see that the maximum core masses attained before the onset of the second dredge-up in the 5 and 7  $M_{\odot}$  sequences are slightly lower than in our models. Thus their larger core masses at the first TP should be a consequence of a less efficient second dredge-up. This is also confirmed by the smaller changes induced by the dredge-up on the surface composition.

Note that B93 even include  $0.7 H_p$  of undershooting in their computations.

## 5. TEST OF THE EVOLUTIONARY SEQUENCES

Although a detailed comparison of our stellar models with the observed properties of different stellar systems is well beyond the purpose of the present paper, let us discuss two interesting tests of the evolutionary sequences recently proposed by different authors, which allow us to check the reliability of the current theoretical scenario. To do that we have computed selected isochrones and synthetic diagrams based on the present set of stellar models.

### 5.1. THE PLEIADES AND THE BROWN DWARF TEST

The certain identification of brown dwarfs would provide important informations on the star formation rate of very low mass stars and contribute to shed some light on the dark matter problem. For this reason, in the last few years, the search of these objects in nearby stellar clusters has been intensified. Brown dwarfs are (quasi) stars for which H burning does not occur or, at least, it does not reach the full equilibrium. How to certificate such an occurrence? A low mass stars approaching the main sequence is fully convective, so that the products of the internal nuclear burning should appear at the surface. Thus, the best indicators of the occurrence of the internal nuclear burning are the secondary elements of the pp-chain. In this context, Li is a good tracer, since it is a very volatile element and it is easily observable in faint objects. Stars with  $M \leq 1M_\odot$  deplete Li even in pre-main sequence. But in very low mass stars, when the internal temperature never exceeds  $2 - 3 \times 10^6$  K, the Li remains unburnt. Therefore, the presence of observable Li lines in stellar spectra is a certain identification of a brown dwarf (Rebolo, Martin & Magazzu 1992). In turn the reappearance

of Li in the lower main sequence of stellar clusters provides an interesting test for the age estimated by means of stellar models. In fact the upper luminosity for which Li is measured depends on the age of the cluster: the larger is the age the fainter is the Li cutoff. Basri et al. (1996) by means of accurate infrared photometry coupled to high resolution spectroscopy of brown dwarfs candidates in the Pleiades, have been able to identify the Li cutoff. By using this Li test they found an age of  $\sim 115$  Myr. They concludes that this value is definitely larger than the age estimated by comparing the turnoff luminosity with that predicted by canonical stellar models.

EDITOR: PLACE FIGURE 21 HERE.

In figure 21 we report the isochrones fitting obtained by using our present stellar models (no overshoot) and that derived by Bertelli et al. (1994) by using the B93 models. Accordingly to the Hypparcos parallaxes, we adopt a true distance modulus of 5.33. A reddening of 0.04 has been assumed. In both cases the isochrones are computed for  $Z=0.02$  and  $Y=0.28$ . Concerning the canonical models, by excluding the isolated bright star at  $V=2.87$ , or  $M_V = -2.58$ , (a blue straggler?), one can get an age of at least 120 Myr and not exceeding 140 Myr, i.e. a value in very good agreement with the brown dwarf test. It is worth noting that owing to the few amount of stars in the turnoff region, a precise age cannot be derived by means of the isochrone fitting. However, even taking into account such an uncertainty, we can exclude ages lower than 100 Myr. Similarly with the Bertelli et al. (1994) isochrones one may get an age in between 150 and 200 Gyr, which is a bit larger then that implied by the brown dwarf Li cutoff. Obviously, also in this case the uncertainty due to the few statistical significance of the number of turnoff stars might be claimed, so that we can't definitely rule out the presence of a moderate overshoot. Let us finally note that this canonical estimation of the Pleiades age, based on the new distance, removes the old controversy of the lack of an evident lower main sequence turnon formed by those stars approaching the ZAMS (Herbig,

1962; Stauffer, 1984). In fact the corresponding lower limit claimed by Stauffer (i.e. 100 Myr) is well in agreement with the present determination.

## 5.2. THE WHITE DWARFS LUMINOSITY FUNCTION

An intermediate mass star ( $M \sim 5 - 6M_{\odot}$ ) ends its life as a CO white dwarf. Thus, if this star is a member of an old stellar system (say 1 Gyr or older), it spent most of its life as a WD so that its cooling time might be used as an age indicator for the stellar system. A search for the cutoff of the WD luminosity function has been recently performed by Von Hippel et al. (1995) by means of the HST with the Wide Field Planetary Camera 2. They were able to identify this cutoff in two old open cluster, namely NGC 2420 and NGC 2477. Then, by means of the theoretical cooling sequences computed by Wood (1994), they estimated the ages of these two clusters and concluded that they are in contrast with all the available stellar models. Owing to the existence of an accurate CCD photometry only for one of these two cluster, namely NGC 2420 (Anthony-Twarog et al 1990), we will focus our attention on this one. From the paper by Von Hippel et al. we derive an age based on the WD luminosity function cutoff of 1.5-1.6 Gyr.

EDITOR: PLACE FIGURE 22 HERE.

In figure 22 we show the isochrones fitting to the Color Magnitude diagram of NGC 2420. According to Anthony-Twarog et al. (1990), we have assumed a metallicity of  $Z=0.008$ , which correspond to about  $[M/H]=-0.4$ , and an  $E(B-V)=0.05$ . Then the distance modulus was derived by reproducing the clump of the He burning stars which is a feature almost independent on the age (see Castellani, Chieffi & Straniero, 1992). The resulting age is of the order of 1.6 ( $\pm 0.2$ ) Gyr, value in very good agreement with the age derived from the WD cooling sequence. A slightly larger value would be obtained by adopting isochrones

including a moderate amount of convective core overshoot. For example, Carraro & Chiosi (1994) found 2.1 Gyr while Friel (1995) reported 2.8 Gyr. However, as already noted by Demarque et al. (1994), the canonical isochrones cannot account for the distribution of stars near the turnoff of NGC 2420. In particular the path of the isochrones just before the overall contraction appears shorter than the observed one. Demarque et al. showed that a moderate amount of convective core overshoot (namely  $\lambda = 0.23H_p$ ) make longer the isochrones path, but they are forced to use an age of 2.4 Gyr.

We argue that the isochrones provide us just the locus "permitted" to the single stars in the CMD. In order to understand if and how this permitted locus is really populated or not (and at what extent) a comparison between observed and synthetic CMD is absolutely required. When a suitable mass function as well as binary stars are considered, a very good reproduction of the observed sequences of NGC 2420 is obtained (see figure 23). In particular the contribution of the binaries leads to a larger spread in the main sequence. Note the effect on the turnoff region: the convective path of single stars seems prolonged by the presence of the binary main sequence and the bluer region after the overall contraction gap is depopulated. Also in this case we obtain an age of 1.6 Gyr which is in very good agreement with the value derived from the WD luminosity function cutoff.

EDITOR: PLACE FIGURE 23 HERE.

## 6. CONCLUSIONS

In this paper we have illustrated the main properties of our latest set of stellar models for intermediate mass stars as obtained by means of the FRANEC code (Chieffi & Straniero 1989). By comparing the most recent evolutionary sequences computed by using different evolutionary codes and/or input physics, we found a rather large disagreement, which is



partially due to the influence of the theoretical assumptions on the estimated extension of the convective regions. We would again remark that not only the difference in the adopted convective algorithm (Schwarzschild criterion, overshooting, semiconvection and the like) is responsible of such a disagreement. The connection among the various ingredients of the model cooking must be understood in order to recognize the origin of the theoretical uncertainties. In some case we found that models obtained by using very different schemes for the treatment of the convective instabilities are more similar than models obtained by using the same algorithm, but different input physics (eos, opacity, nuclear reaction rate and the like).

In spite of this disagreement, since the brighter is a model the lower is the lifetime, many differences are smoothed away when transposing the evolutionary tracks into the isochrones. This has been already shown in the previous section where we compare our theoretical isochrones and those by Bertelli et al (1994) with the Color Magnitude diagrams of some well studied Open Clusters (see figure 21). Although the evolutionary features of these two sets of models are rather different, the resulting isochrone fittings are quite similar in the two cases. This means that a quite similar isochrone path may be obtained simply by rescaling the mass (or the age). Such an occurrence is clearly illustrated in the example reported in figure 24. In this figure we compare our isochrones of 0.7 Gyr with the ones of Bertelli et al having 0.8 Gyr.

EDITOR: PLACE FIGURE 24 HERE.

We recall that the Bertelli isochrones were obtained by assuming a moderate amount of overshooting, whereas our models do not include any extra mixing with respect to the instability boundary. As discussed in the previous section, the only evident difference is in the shape of the turnoff.

Another quantities which is well established in the framework of the current theory of the stellar evolution is the core mass attained at the beginning of the AGB phase. In fact, since the luminosity of an off main sequence star is mainly controlled by the size of its He-core mass, the H-burning shell have more time to advance in mass in models with lower core mass,

Let us finally comment that the use of the Color Manitude diagrams to check the reliability of a particular set of models cannot be barely made by means of the isochrone fitting. In fact the best photometric studies of open clusters include few thousand of stars. Then the "permitted" locus do not necessarily coincide with the "populated" locus. In addition many Open Clusters have a huge population of binary stars which contribute to determine the shape of the observed Color Magnitude diagrams. The case of NGC2420, as discussed in the previous section, is a template of such a situation.

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## FIGURE CAPTIONS

Fig. 1.— Evolutionary tracks for  $Z=0.02$   $Y=0.28$

Fig. 2.— Evolutionary tracks for  $Z=0.006$   $Y=0.26$

Fig. 3.— Evolutionary tracks for  $Z=0.001$   $Y=0.23$

Fig. 4.— Evolutionary tracks for  $Z=0.0001$   $Y=0.23$

Fig. 5.— The evolution of the central temperature versus the central density for  $Z=0.02$  and  $Y=0.28$ .

Fig. 6.— The evolution of the central temperature versus the central density for  $Z=0.006$  and  $Y=0.26$ .

Fig. 7.— The evolution of the central temperature versus the central density for  $Z=0.001$  and  $Y=0.23$ .

Fig. 8.— The evolution of the central temperature versus the central density for  $Z=0.0001$  and  $Y=0.23$ .

Fig. 9.— The evolution of the convective regions (hashed areas) for the  $2.5 M_{\odot}$  of solar chemical composition.

Fig. 10.— The evolution of the convective regions (hashed areas) for the  $4 M_{\odot}$  of solar chemical composition.

Fig. 11.— The evolution of the convective regions (hashed areas) for the  $7 M_{\odot}$  of solar chemical composition.

Fig. 12.— The evolution of the convective regions (hashed areas) for the  $9 M_{\odot}$  of solar



chemical composition.

Fig. 13.— The He core mass at the He ignition as a function of the total mass.

Fig. 14.— He-burning lifetime versus the stellar mass for  $Z=0.02$  and  $Y=0.28$ .

Fig. 15.— The He core mass at the beginning of the TP-AGB phase as a function of the total mass.

Fig. 16.— The final masses: the He core masses at the beginning of the TP-AGB phase (squares); the residual masses at the end of the AGB (triangles); initial/final mass relation by Weideman (1987) for the galactic disk (dotted line) and for the Magellanic Clouds (dashed line); the initial/final mass relation updated by Herwig (1995) for the galactic disk (solid line). The numeric labels indicate the number of thermal pulses occurring before the end of the AGB phase.

Fig. 17.— Comparison among the present evolutionary tracks and those presented by Castellani, Chieffi & Straniero (1990 and 1992).

Fig. 18.— Comparison among the present H-burning lifetimes (pp) and the those by Bressan et al (1993, B93) and Shaller et al. (1992, S92). The differences (in percent) for each masses are reported.

Fig. 19.— As in figure 18, but for the He-burning lifetimes.

Fig. 20.— Comparison among various theoretical core masses at the end of the E-AGB: present paper (solid line), Lattanzio (1991)  $Y=0.20$  (squares), Lattanzio (1991)  $Y=0.30$  (triangles), Bressan et al. (1993) (circles).

Fig. 21.— Isochrones fitting to the Pleiades by using our isochrones (left panel) and those of Bertelli et al. (1994, right panel).

Fig. 22.— Isochrones fitting to NGC2420.

Fig. 23.— True and synthetic CMDs. Upper-left panel: the observed CMD of NGC2420. Other three panels: synthetic CMDs as obtained under different assumption about the exponent ( $\alpha$ ) of the mass function. In computing these synthetic diagrams we have assumed a 30% of binary stars and an age of 1.6 Gyr.

Fig. 24.— Comparison between our isochrones for an age of 0.7 Gyr (no overshoot) and the one by Bertelli et al. (1994) for 0.8 Gyr (moderate overshoot). Both these isochrones have  $Z=0.02$  and  $Y=0.28$ .